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A Computational General Equilibrium Model with Vintage Capital

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ABSTRACT

This paper presents a vintage capital model assuming putty-clay investment and perfect foresight. The model is written in discrete time and is simulated by using a second order relaxation algorithm. By computing the eigenvalues of the dynamic system, we have first checked the conditions of existence and uniqueness of a solution (Blanchard and Khan's conditions) and identified the echo effect that characterizes vintage capital models and the related dynamics of creation and destruction. By calibrating the model on French data, it has been proved useful to explain the medium-term movements in the distribution of income in France during the last three decades.

Keywords : Vintage capital models, replacement echoes, dynamic model solving, medium-term dynamics.

JEL classification : C68, C63, E22.

RÉSUMÉ

Ce document présente un modèle à génération de capital reposant sur des anticipations parfaites et une technologie de production *putty-clay*. Bien que l'utilisation de générations d'unités de production est une façon raisonnable de modéliser la dynamique de l'offre agrégée, cette méthode pose de nombreux problèmes de calcul qui expliquent pourquoi elle a été si peu populaire dans la littérature économique. En effet, avec une fonction de production *putty-clay*, l'inconvénient principal réside dans l'inclusion dans le modèle de variables présentant de nombreux retards et de nombreuses avances. Par exemple, la durée de vie de chaque unité de production est déterminée en fonction de la profitabilité anticipée sur une longue période de temps. Ainsi, toute modélisation convenable de ce type de modèle a souvent été considérée comme trop lourde.

Depuis le milieu des années 90, à partir des travaux de Caballero et Hammour (1994) et Boucekkine et al. (1997), ce champ de recherche a connu un regain d'intérêt en raison de sa capacité à expliquer les principales évolutions économiques observées dans les pays industrialisés au cours des trois dernières décennies. Les technologies *putty-clay* impliquant une certaine rigidité dans le processus de production, ces types de travaux ont permis d'étudier convenablement l'ajustement lent des facteurs de production à des chocs. Ce cadre d'analyse a également permis de tenir compte de manière explicite des évolutions en terme de création et de destruction d'emploi liées à l'obsolescence économique, au remplacement des capacités productives et aux anticipations sur la durée de vie des nouvelles unités de production.

L'originalité du modèle proposé ici est son écriture en temps discret alors que les autres travaux récents développent au contraire des modèles en temps continu. Pour surmonter les problèmes de calculs habituellement présents dans ce type de modélisation, nous utilisons l'algorithme Stack développé pour résoudre des modèles à anticipations rationnelles de grande taille.

Parce que le modèle est en temps discret, il est possible de calculer les valeurs propres du système dynamique. Ceci est utile, tout d'abord, pour vérifier les conditions d'existence et d'unicité d'une solution (conditions de Blanchard et Khan). Surtout, l'analyse des valeurs propres améliore la compréhension des différentes dynamiques à l'œuvre dans l'économie. En particulier, nous pouvons identifier l'effet d'écho qui caractérise les modèles à génération de capital et leurs dynamiques associées en terme de création et de destruction d'emploi.

Le modèle s'avère également utile pour expliquer les évolutions à moyen terme de la distribution des revenus entre facteurs de production, ce que les modèle à technologie *putty-putty* sont incapables de faire. En particulier, notre modèle illustre assez bien la modification de la part des salaires dans la valeur ajoutée en France au cours des trois dernières décennies.

SUMMARY

This paper presents a vintage capital model assuming putty clay investment and perfect foresight. Although using vintage units of production is a sensible way to model the dynamics of aggregate supply, it raises many computational problems that explain why it has not been very popular in the economic literature. Indeed, with a putty-clay production function, the traditional drawback is the presence of variables with long leads and long lags. For instance, the lifetime of each vintage unit of production is set according to the expected profitability over a long time period. Thus, any proper modeling of this type of model has often been considered as too cumbersome.

Since the mid 90s, from the works by Caballero and Hammour (1994) and Boucekkine et al. (1997), this research field has known a renewed interest for its ability to explain some major economic developments observed in industrialized countries over the last three decades. First, as putty-clay technology involves some stickiness in the production process, such works have been able to investigate properly the slow adjustment of production factors to shocks. Second, this framework also explicitly takes into account movements in job creation and job destruction related to economic obsolescence, replacement of productive capacity and expectations over the lifetime of the new units of production.

The originality of the model proposed here is that it is in discrete time whereas previous, recent works have developed model in continuous time. To overcome the traditional computational restrictions, we use the stack-algorithm developed to solve large rational expectation models.

With such a discrete time model, it is possible to compute the eigenvalues of the dynamic system. First, this is useful to check the conditions of existence and uniqueness of a solution (Blanchard and Khan's conditions). More importantly, the analysis of the eigenvalues improves the understanding of the different dynamics in the economy. In particular, we can identify the echo effect that characterizes vintage capital models and the related dynamics of job creation and job destruction.

This model is also proved useful to explain the medium-term movements in the distribution of income between production factors that putty-putty models lack. In particular, it illustrates quite well the change in the wage share in value-added in France during the last three decades.

A Computational General Equilibrium Model with Vintage Capital[1]

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I - INTRODUCTION

Computational general equilibrium models usually assume a putty-putty technology: the capital intensity of the production process can be changed instantaneously and without cost. Thus, in a competitive framework, the factors of production fully and instantaneously adjust to current economic conditions. This means that “realistic” changes in real wages or in the cost of capital lead to very significant and quick moves in demand for labor and capital. Moreover, the quick adjustment of the capital stock should cause huge variations in the flows of investment.

However, actual employment and capital stock exhibit much weaker movements than those predicted above. Hence, the integration of this theoretical framework in a realistic model requires some improvements. One way to decrease the cost-sensitivity of production factors consists in assuming nonlinear adjustment costs (usually quadratic costs). This results in smoother dynamic adjustments of labor and capital. However, this specification rests upon an *ad hoc* assumption without wholly rigorous microeconomic foundations and empirical verification. Moreover, it is not a fully convincing way to model the irreversibility of investment and the firing costs of labor. Finally, the putty-putty framework is unable to give simple, acceptable explanations for the medium term movements in the wage share in value added, which are observed in some European countries (see for instance Blanchard, 1997, Prigent, 1999). Although adjustment costs smooth the dynamics of factor demands in the short run, they are far from sufficient to produce medium term changes in the income distribution between capital and labor.

A key feature of the putty-putty specification, that is central to its empirical failure, is that all the vintages of capital have the same capital intensity. On the contrary, we would expect the current technology menu to be only available to the newly created units of production. This is precisely what the putty-clay specification does. In this framework, current economic conditions affect the capital intensity of the new production units (their technological choice) and the number of these units created (investment in the economy). The other production units keep the technology they were given at their creation. However,

[1] The ideas developed here were inspired by a series of papers by Caballero and Hammour. We have benefited of very useful comments by Agnès Bénassy, Pierre-Yves Hénin, Paul Zagamé, Pierre Sicsic and Werner Roeger. Stéphanie Guichard made invaluable criticisms and suggestions on previous versions of the paper.

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current economic conditions affect their profitability and lead to the scrapping of non-profitable units. Hence, the aggregate capital-labor ratio changes gradually with the flows of investment and the scrapping of old obsolete production units. Putty-clay investment may thus provide medium term dynamics in the distribution of income.

This specification has some other advantages. The irreversibility of investment is embedded in the model and firing costs can easily be introduced. This gives a convincing foundation to the stickiness of employment.

Despite all its advantages, the putty-clay technology suffers from a serious drawback. Its implementation in a macroeconomic model is cumbersome for two reasons. First, the model has a long memory since it keeps track of all the vintages of capital created in the past, that are still in working order. Thus, the model has "variables with long lags". Second, the planning horizon of investors stretches far into the future. More precisely, the decision concerning the new production units involves forward variables that cover the expected lifetime of these units. The model has then "variables with long leads".

However, these problems can be easily overcome nowadays. Models with variables presenting long leads and lags can be solved with powerful algorithms (for instance those implemented in Troll), and simulation time is decreasing with the improvement of personal computers.

The first section presents a model representing the production of goods and the demand of factors with a putty-clay technology. In the second section, we close the model by completing its demand side, by introducing a "wage curve" and by assuming the equilibrium of the goods market. Then, we describe the determination of the equilibrium. The third section presents the results of the simulation of the model. The calibration is such that the steady state of the model is identical to the situation of the French economy for the year 1994. Then, we investigate whether the properties of existence and uniqueness of a solution to the model are satisfied. Finally, we identify a dampened replacement echo effect, with a length equal to the lifetime of a production unit on the steady state. The last section discusses the consequences of an unanticipated permanent change in the wage-setting relationship in France. It also shows the ability of this framework to replicate the medium term changes in the distribution of income that France experienced during the last three decades.

II - TECHNOLOGY AND FACTORS DEMANDS

In this section, we introduce the specification of the technology of firms and we determine their decisions. At each date, firms build a number of new production units, which will start to produce one period later. They have to choose the capital intensity embodied in these units. Since this capital intensity cannot change in the future, the expected lifetime of the new production units is part of the decision-making. Both capital intensity and expected lifetime are set to maximize the expected discounted cash flows minus installation costs. Moreover, firms reassess the profit that each older unit would make during the current period if it were kept into activity. When this profit is negative the unit is scrapped. Then, we can aggregate the decisions of firms and get the investment, the employment and the production of the current period.

2.1. Technology and investment cost

We consider a representative firm, on a perfectly competitive goods market, which must make choices at time t_0 [2]. At this time, the firm decides to acquire k_{t_0} new units of capital. It also chooses the technology embodied in this capital. The technology menu is characterized by an *ex ante* first-order homogenous production function:

$$F(k_{t_0}, A_{t_0} n_{t_0}) = z[a k_{t_0}^{1-1/s} + (1-a)(A_{t_0} n_{t_0})^{1-1/s}]^{s/(s-1)}, \text{ with: } z, s > 0, 0 < a < 1$$

This equation determines the amount of goods produced by combining k_{t_0} units of capital in equipment with n_{t_0} units of labor. A_{t_0} represents the efficiency of the technology available at time t_0. s is the *ex ante* elasticity of factor substitution. The capital intensity $\mathbf{k}_{t_0}$ chosen at time t_0 for the entire lifetime of the capital units is defined by:

$$\mathbf{k}_{t_0} = k_{t_0} / A_{t_0} n_{t_0}$$

The production function can be rewritten as:

$$F(\mathbf{k}_{t_0}, 1) = z[a \mathbf{k}_{t_0}^{1-1/s} + (1-a)]^{s/(s-1)} \quad (1)$$

In the future, the vintage of capital created at t_0 will either be used with capital intensity $\mathbf{k}_{t_0}$ or be scrapped.

We define one unit of production created at time t_0 as the combination of one unit of labor and $\mathbf{k}_{t_0} A_{t_0}$ units of capital. This unit of production produces $A_{t_0} F(\mathbf{k}_{t_0}, 1)$ units of goods. At the beginning of period t_0, i.e. at time t_0, n_{t_0} new units of production are created. At the end of each period, a fraction d of the firms goes bankrupt and stops producing. d can be seen as the rate at which the production units disappear for all reasons but macroeconomic conditions (bad management, mistakes or technical difficulties in the implementation of production). In that sense, d is equivalent to an exogenous depreciation rate.

For every unit, production starts in the period following its installation. The units created at time t_0 are productive from the beginning of period $t_0 + 1$, i.e. at time $t_0 + 1$.

Aggregate investment in equipment at time t_0 is defined by:

[2] As firms are identical, the choices made by any individual firm also hold at the aggregate level. Time t_0 represents the beginning of period t_0.

$$I_{t_0} = A_{t_0} k_{t_0} n_{t_0} \tag{2}$$

The cost of one unit of capital, including the installation costs, expressed in units of good produced, is exogenous and equal to $c_{i_{t_0}} > 0$.

2.2. Value of a new production unit

We define r_t as the nominal interest rate of an asset with a maturity of one period, available at time t. We also define $w_t A_t$ as the nominal wage paid to each worker of the production unit for the work done during period t. By convention, we assume that this wage is paid at the end of period t. Let us consider a unit of production created at time t_0. When capital is scrapped, at time $t_0 + T(t_0)$, firing the workers costs $p_{t_0+T(t_0)} x^f_{t_0+T(t_0)} A_{t_0+T(t_0)}$ where $p_{t_0+T(t_0)}$ is the price level at time $t_0 + T(t_0)$ and $x^f_{t_0+T(t_0)}$ is the real firing cost in efficiency unit[3]. If the production unit goes bankrupt, however, no firing cost is incurred.

We define by V_{t,t_0} the present value of the cash flows of the production unit built at t_0, measured at the end of period t, i.e. the value of the production unit for the firms that have not gone bankrupt until that time. In case of bankruptcy, this value is zero.

We have the following arbitrage condition:

$$(1-d)V_{t+1,t_0} + (1-tr_{t+1})\, p_{t+1} A_{t_0} F(k_{t_0},1) - w_{t+1} A_{t+1} = (1+r_t)V_{t,t_0}$$

where tr is the tax rate on production.

At the end of period *t*, the owner of a production unit can either sell it and invest the proceeds in the financial markets or hold it. In the first case, it will get $(1+r_t)V_{t,t_0}$ at the end of period *t+1*. In the second case, the unit will yield an after tax profit at the end of period *t+1* equal to: $(1-tr_{t+1})\, p_{t+1} A_{t_0} F(k_{t_0},1) - w_{t+1,t_0} A_{t+1}$. Besides, at the end of period *t+1*, the unit will either go bankrupt, with probability d, or still be in working order, with probability ($1-d$).

It is now possible to define an initial condition and a terminal condition for the first difference equation above.

[3] This means that the unit of production has been active from the beginning of period $t_0 + 1$ to time $T(t_0)$, that is for $T(t_0) - 1$ periods.

First, the assumption of free entry means that the financial value of a new production unit at the end of the period of creation is equal to its building cost (taking into account the possibility of bankruptcy):

$$(1-d)V_{t_0,t_0} = p_{t_0} c_{i_{t_0}} A_{t_0} k_{t_0}$$

The investor who decides to create a new production unit pays its building cost during the installation period t_0 : $p_{t_0} c_{i_{t_0}} A_{t_0} k_{t_0}$. He is aware that this unit has a probability d to go bankrupt at the end of period t_0, and a probability $(1-d)$ to remain in working order. Its value at the end of period t_0, expected at the beginning of period t_0 is then: $(1-d)V_{t_0,t_0}$.

Second, when the production unit is scrapped, the owner has to pay for the firing costs. Note that the expected scrapping date of the new unit, $t_o + T(t_o)$, is not necessarily an integer number of years. We define $\overline{T}(t_o)$ as the integer part of the expected lifetime of a new production unit, and $\Delta T(t_o)$ as its decimal part[4]. The value of the production unit, at the date it is scrapped, is equal to the firing costs. This defines the terminal condition.

In the model, the production units created at time t_0 will be productive for $\overline{T}(t_o)$ full periods of time, i.e. from the beginning of period t_0+1 until the end of period $t_0+\overline{T}(t_0)$. It will still be in working order at the beginning period $t_0+\overline{T}(t_0)+1$, but will be only productive during a fraction of period equal to $\Delta T(t_0)$. Hence, to write down the terminal condition, we must make additional assumptions on the value of nominal wage (in efficiency unit), price and firing costs at the date $t_0+\overline{T}(t_0)+1+\Delta T(t_0)$. We assume that these three variables at this date are equal to their geometrical interpolations between (the end of period) $t_0+\overline{T}(t_0)$ and (the end of period) $t_0+\overline{T}(t_0)+1$:

$$w_{t_0+\overline{T}(t_0)+1+\Delta T(t_0)} = w_{t_0+\overline{T}(t_0)}{}^{1-\Delta T(t_0)} \, w_{t_0+\overline{T}(t_0)+1}{}^{\Delta T(t_0)}$$

$$p_{t_0+\overline{T}(t_0)+1+\Delta T(t_0)} = p_{t_0+\overline{T}(t_0)}{}^{1-\Delta T(t_0)} \, p_{t_0+\overline{T}(t_0)+1}{}^{\Delta T(t_0)}$$

[4] The expected lifetime corresponds to the period of time during which a unit is in working order. Thus, we have $T(t_0) = \overline{T}(t_o) + 1 + \Delta T(t_0)$. All the units of the same vintage are identical. We want to avoid scrapping choices to be discrete: at a given integer time, all the units of a given vintage are either scrapped or kept in use. With a model using the year as basic time unit, this discrete choice would introduce too strong discontinuities in the response of the model to shocks. A better assumption is that part of a given vintage is scrapped at the beginning of a period, and the rest is scrapped at the beginning of next period. The assumption, made in the paper is that a vintage is scrapped at an intermediary time inside a period. This assumption is slightly different, but a little more practical.

$$x^{f}_{t_0+\overline{T}(t_0)+1+\Delta T(t_0)} = x^{f\;\;1-\Delta T(t_0)}_{t_0+\overline{T}(t_0)} x^{f\;\;\Delta T(t_0)}_{t_0+\overline{T}(t_0)+1}$$

We assume that the income related to the fraction of the period starting at the beginning of period $t_0+\overline{T}(t_0)+1$ and lasting $\Delta T(t_0)$ is received at time $t_0+\overline{T}(t_0)+1+\Delta T(t_0)$ and taxed at the rate of the period $t_0+\overline{T}(t_0)+1$. Its value at the end of period $t_0+\overline{T}(t_0)$ is obtained using the discounted factor $(1+r_{t_o+\overline{T}(t_o)})^{\Delta T(t_o)}$. We also assume that the probability not to go bankrupt during the fraction of time $\Delta T(t_0)$ is equal to: $(1-d)^{\Delta T(t_o)}$. Finally, technical progress is assumed to be a continuous function of time given by: $A(t_0+t)=A(t_0)(1+\gamma)^t$ for $t \geq 0$.

We now have the terminal condition:

$$\begin{aligned} &V_{t_0+T(t_0),t_0} = \\ &+ A_{t_0}\Delta T(t_0)\begin{bmatrix} (1-tr_{t_0+\overline{T}(t_0)+1})\, p_{t_0+\overline{T}(t_0)}^{\;1-\Delta T(t_0)}\, p_{t_0+\overline{T}(t_0)+1}^{\;\Delta T(t_0)} F(\kappa_{t_0},1) \\ - w_{t_0+\overline{T}(t_0)}^{\;1-\Delta T(t_0)}\, w_{t_0+\overline{T}(t_0)+1}^{\;\Delta T(t_0)}(1+\gamma)^{T(t_0)} \end{bmatrix} \\ &- p_{t_0+\overline{T}(t_0)}^{\;1-\Delta T(t_0)}\, p_{t_0+\overline{T}(t_0)+1}^{\;\Delta T(t_0)}\left[x^{f\;\;1-\Delta T(t_0)}_{t_0+\overline{T}(t_0)}\, x^{f\;\;\Delta T(t_0)}_{t_0+\overline{T}(t_0)+1}\right] A_{t_0}(1+\gamma)^{T(t_0)} \end{aligned}$$

The value of the firm when it is created is the present value of future expected profits minus the firing costs at the expected scrapping age[5].

The value of a new production unit is obtained by summing the arbitrage equation until its scrapping date:

[5] We could assume instead that the creation of a new production unit at the beginning of period t_0 enables firms to save a fraction of the firing costs incurred by scrapping units at the beginning of period t_0+1. Part of the labor, which is in the working or the scrapped units, would be transferred to the new units.

$$V_{t_0,t_0} = A_{t_0} \sum_{s=t_0+1}^{t_0+\overline{T}(t_0)} \left[(1-tr_s)p_s F(\kappa_{t_0},1) - w_s(1+\gamma)^{s-t_0}\right](1-\delta)^{s-t_0-1} / \left(\prod_{\tau=t_0}^{s-1}(1+r_\tau)\right)$$

$$+ p_{t_0+\overline{T}(t_0)} A_{t_0} \left(\frac{p_{t_0+\overline{T}(t_0)+1}}{p_{t_0+\overline{T}(t_0)}} \left(\frac{1-\delta}{1+r_{t_0+\overline{T}(t_0)}} \right) \right)^{\Delta T(t_0)}$$

$$\Big\{\Delta T(t_0)\Big[(1-tr_{t_0+\overline{T}(t_0)+1})F(\kappa_{t_0},1) - \left(w_{t_0+\overline{T}(t_0)} / p_{t_0+\overline{T}(t_0)}\right)^{1-\Delta T(t_0)} \left(w_{t_0+\overline{T}(t_0)+1} / p_{t_0+\overline{T}(t_0)+1}\right)^{\Delta T(t_0)} (1+\gamma)^{T(t_0)}\Big]$$

$$- {x^f_{t_0+\overline{T}(t_0)}}^{1-\Delta T(t_0)} {x^f_{t_0+\overline{T}(t_0)+1}}^{\Delta T(t_0)} (1+\gamma)^{T(t_0)} \Big\} (1-\delta)^{\overline{T}(t_0)-1} / \left(\prod_{\tau=t_0}^{t_0+\overline{T}(t_0)-1} (1+r_\tau) \right)$$

2.3. Characteristics of the new production units

When a new unit is created, its owners must choose the technology embodied in it on the basis of its planned lifetime, and of the market conditions they expect during that period. The choices of the planned lifetime ($T(t_0)$) and of the capital intensity of this new unit (k_{t_0}) result from the maximization of the value of this unit minus its installation cost:

$$\Psi_{t_0}\left(\overline{T}(t_0, \Delta T(t_0), \kappa_{t_0}\right) = (1-\delta)V_{t_0,t_0} - p_{t_0} c_{i_{t_0}} A_{t_0} \kappa_{t_0} .$$

<u>Expected life time</u>

The integer part of the expected lifetime, $\overline{T}(t_o)$, is defined by:

$$\begin{cases} \Psi_{t_0}\left(\overline{T}(t_0), \Delta T(t_0), \kappa_{t_0}\right) - \Psi_{t_0}\left(\overline{T}(t_0)-1, \Delta T(t_0), \kappa_{t_0}\right) > 0 \\ \Psi_{t_0}\left(\overline{T}(t_0)+1, \Delta T(t_0), \kappa_{t_0}\right) - \Psi_{t_0}\left(\overline{T}(t_0), \Delta T(t_0), \kappa_{t_0}\right) < 0 \end{cases} \quad (3)$$

The decimal part of the expected scrapping date of the unit $\Delta T(t_o)$ is determined by the following first-order condition:

$$\frac{\partial \Psi_{t_0}(\overline{T}(t_0), \Delta T(t_0), \kappa_{t_0})}{\partial \Delta T(t_0)} = 0$$

Thus:

$$
\begin{aligned}
&\ln\left(\frac{p_{t_0+\bar{T}(t_0)+1}}{p_{t_0+\bar{T}(t_0)}}\left(\frac{1-d}{1+r_{t_0+\bar{T}(t_0)}}\right)\right)\\
&\left\{\begin{array}{l}\Delta T(t_0)\left((1-tr_{t_0+\bar{T}(t_0)+1})(1+g)^{-T(t_0)}F(k_{t_0},1)-\left(\frac{w_{t_0+\bar{T}(t_0)}}{p_{t_0+\bar{T}(t_0)}}\right)^{1-\Delta T(t_0)}\left(\frac{w_{t_0+\bar{T}(t_0)+1}}{p_{t_0+\bar{T}(t_0)+1}}\right)^{\Delta T(t_0)}\right)\\
-{x^f_{t_0+\bar{T}(t_0)}}^{1-\Delta T(t_0)}{x^f_{t_0+\bar{T}(t_0)+1}}^{\Delta T(t_0)}\end{array}\right\}\\
&+\left((1-tr_{t_0+\bar{T}(t_0)+1})(1+g)^{-T(t_0)}F(k_{t_0},1)-\left(\frac{w_{t_0+\bar{T}(t_0)}}{p_{t_0+\bar{T}(t_0)}}\right)^{1-\Delta T(t_0)}\left(\frac{w_{t_0+\bar{T}(t_0)+1}}{p_{t_0+\bar{T}(t_0)+1}}\right)^{\Delta T(t_0)}\right)\\
&-\Delta T(t_0)\left(\frac{w_{t_0+\bar{T}(t_0)}}{p_{t_0+\bar{T}(t_0)}}\right)^{1-\Delta T(t_0)}\left(\frac{w_{t_0+\bar{T}(t_0)+1}}{p_{t_0+\bar{T}(t_0)+1}}\right)^{\Delta T(t_0)}\ln\left(\frac{w_{t_0+\bar{T}(t_0)+1}/p_{t_0+\bar{T}(t_0)+1}}{w_{t_0+\bar{T}(t_0)}/p_{t_0+\bar{T}(t_0)}}\right)\\
&-{x^f_{t_0+\bar{T}(t_0)}}^{1-\Delta T(t_0)}{x^f_{t_0+\bar{T}(t_0)+1}}^{\Delta T(t_0)}\ln\left(\frac{x^f_{t_0+\bar{T}(t_0)+1}}{x^f_{t_0+\bar{T}(t_0)}}\right)\\
&-\left(\Delta T(t_0)\left(\frac{w_{t_0+\bar{T}(t_0)}}{p_{t_0+\bar{T}(t_0)}}\right)^{1-\Delta T(t_0)}\left(\frac{w_{t_0+\bar{T}(t_0)+1}}{p_{t_0+\bar{T}(t_0)+1}}\right)^{\Delta T(t_0)}+{x^f_{t_0+\bar{T}(t_0)}}^{1-\Delta T(t_0)}{x^f_{t_0+\bar{T}(t_0)+1}}^{\Delta T(t_0)}\right)\ln(1+g)=0
\end{aligned}
\tag{4}
$$

In the neighborhood of the steady state and for small growth rates, (3) and (4) determine a unique integer value for $\bar{T}(t_0)$ and the real value included between 0 and 1 for $\Delta T(t_0)$ (proof available on request).

<u>Capital intensity</u>

The capital intensity chosen by investors for the new production units is given by the first-order condition:

$$
\frac{\partial \Psi_{t_0}(\bar{T}(t_0), \Delta T(t_0), \kappa_{t_0})}{\partial \kappa_0} = 0
$$

$$
\begin{aligned}
&F_1'(k_{t_0},1)\left\{\sum_{s=t_0+1}^{t_0+\bar{T}(t_0)} p_s(1-tr_s)(1-d)^{s-t_0-1}\Big/\prod_{t=t_0}^{s-1}(1+r_t)\right.\\
&\left.+\Delta T(t_0)\,p_{t_0+\bar{T}(t_0)}\left(\frac{p_{t_0+\bar{T}(t_0)+1}}{p_{t_0+\bar{T}(t_0)}}\left(\frac{1-d}{1+r_{t_0+\bar{T}(t_0)}}\right)\right)^{\Delta T(t_0)}(1-tr_{t_0+\bar{T}(t_0)+1})(1-d)^{\bar{T}(t_0)-1}\Big/\prod_{t=t_0}^{t_0+\bar{T}(t_0)-1}(1+r_t)\right\}\\
&=p_{t_0}c_{i_{t_0}}/(1-d)
\end{aligned}
\tag{5}
$$

where $F_1^{'}(k,1)$ is the marginal productivity of capital.
By combining this equation and the free-entry condition, we get (proof available on request):

$$\begin{aligned}
&\sum_{s=t_0+1}^{t_0+\overline{T}(t_0)} \left[(1-tr_s)p_s F_2^{'}(\kappa_{t_0},1) - w_s(1+\gamma)^{s-t_0}\right](1-\delta)^{s-t_0-1} \Big/ \left(\prod_{\tau=t_0}^{s-1}(1+r_\tau)\right) \\
&+ p_{t_0+\overline{T}(t_0)} A_{t_0} \left(\frac{p_{t_0+\overline{T}(t_0)+1}}{p_{t_0+\overline{T}(t_0)}} \left(\frac{1-\delta}{1+r_{t_0+\overline{T}(t_0)}}\right)\right)^{\Delta T(t_0)} \qquad (6) \\
&\left\{\Delta T(t_0)\left[\begin{array}{l}(1-tr_{t_0+\overline{T}(t_0)+1})F_2^{'}(\kappa_{t_0},1) - \left(w_{t_0+\overline{T}(t_0)}/p_{t_0+\overline{T}(t_0)}\right)^{1-\Delta T(t_0)} \left(w_{t_0+\overline{T}(t_0)+1}/p_{t_0+\overline{T}(t_0)+1}\right) \\ {}^{\Delta T(t_0)}(1+\gamma)^{T(t_0)}\end{array}\right]\right. \\
&\left. - x^f_{t_0+\overline{T}(t_0)}{}^{1-\Delta T(t_0)} x^f_{t_0+\overline{T}(t_0)+1}{}^{\Delta T(t_0)} (1+\gamma)^{T(t_0)}\right\}(1-\delta)^{\overline{T}(t_0)-1} \Big/ \left(\prod_{\tau=t_0}^{t_0+\overline{T}(t_0)-1}(1+r_\tau)\right) = 0
\end{aligned}$$

If capital intensity is eliminated between equations (5) and (6), we get a factor cost frontier which involves present and future interest rates and future wages rates.

Boucekkine, Germain, Licandro and Magnus (2000) investigate the best way to solve numerically a vintage capital model. They start with a model written in continuous time and advise to approximate it in discrete time. We prefer to start with a model written in discrete time and to make intra-period approximations. They solve the discrete optimization problem directly by an iterative method. We prefer to write the first-order conditions, and to simulate the equations of the model including these conditions.

2.4. Scrapping and aggregation

We now consider the decisions concerning the production units built before time t_0. For each vintage of capital, its capital intensity being already set, the investor checks whether it is still profitable. If not, it is discarded.
Under our assumptions, a production unit is built during a period (which is an integer number), but can be used for the whole or part of a period. Let us call $\overline{a}(t_0)$ the age of the oldest unit which is used at the beginning of period t_0 [6], i.e. at time t_0, and which will be scrapped at time $t_0 + \Delta a(t_0) < t_0 + 1$. This unit was built at period $t_0 - \overline{a}(t_0)$. Its value at the end of this period is [7]:

---

[6] So, the unit has been active at this date for $\overline{a}(t_0) - 1$ periods.

[7] Wage, price and firing costs within period t_0 are geometrical interpolations of their values in (the end of) periods $t_0 - 1$ and t_0.

$$V_{t_0-\overline{a}(t_0),t_0-\overline{a}(t_0)}(\overline{a}(t_0)+\Delta a(t_0)-1)=$$
$$A_{t_0-\overline{a}(t_0)}\sum_{s=t_0-\overline{a}(t_0)+1}^{t_0-1}\left[(1-tr_s)p_s F(\kappa_{t_0-\overline{a}(t_0)},1)-w_s(1+\gamma)^{s-t_0+\overline{a}(t_0)}\right]$$
$$(1-\delta)^{s-t_0+\overline{a}(t_0)-1}/\left(\prod_{\tau=t_0-\overline{a}(t_0)}^{s-1}(1+r_\tau)\right)$$
$$+A_{t_0-\overline{a}(t_0)}p_{t_0-1}\left(\frac{p_{t_0}}{p_{t_0-1}}\left(\frac{1-\delta}{1+r_{t_0-1}}\right)\right)^{\Delta a(t_0)}$$
$$\left\{\Delta a(t_0)\left((1-tr_{t_0})F(\kappa_{t_0-\overline{a}(t_0)},1)-\left(w_{t_0-1}/p_{t_0-1}\right)^{1-\Delta a(t_0)}\left(w_{t_0}/p_{t_0}\right)^{\Delta a(t_0)}(1+\gamma)^{\overline{a}(t_0)+\Delta a(t_0)-1}\right)\right.$$
$$\left.-x_{t_0-1}^{f\ \ 1-\Delta a(t_0)}x_{t_0}^{f\ \Delta a(t_0)}(1+\gamma)^{\overline{a}(t_0)+\Delta a(t_0)-1}\right\}(1-\delta)^{\overline{a}(t_0)-2}/\left(\prod_{\tau=t_0-\overline{a}(t_0)}^{t_0-2}(1+r_\tau)\right)$$

Let us define:

$$\Psi_{t_0-\overline{a}(t_0)}\left(\overline{a}(t_0)-1,\Delta a(t_0),\kappa_{t_0-\overline{a}(t_0)}\right)=(1-\delta)V_{t_0-\overline{a}(t_0),t_0-\overline{a}(t_0)}$$
$$-\ p_{t_0-\overline{a}(t_0)}c_{i\ t_0-\overline{a}(t_0)}A_{t_0-\overline{a}(t_0)}\kappa_{t_0-\overline{a}(t_0)}$$

The age of the oldest production units used at the beginning of period t_0 is defined by:

$$\begin{cases}\Psi_{t_0-\overline{a}(t_0)}(\overline{a}(t_0)-1,\Delta a(t_0),\kappa_{t_0-\overline{a}(t_0)})-\Psi_{t_0-\overline{a}(t_0)}(\overline{a}(t_0)-2,\Delta a(t_0),\kappa_{t_0-\overline{a}(t_0)})>0\\ \Psi_{t_0-\overline{a}(t_0)}(\overline{a}(t_0),\Delta a(t_0),\kappa_{t_0-\overline{a}(t_0)})-\Psi_{t_0-\overline{a}(t_0)}(\overline{a}(t_0)-1,\Delta a(t_0),\kappa_{t_0-\overline{a}(t_0)})<0\end{cases}\quad (7)$$

The period of time during which the oldest units are in working order within period t_0 is defined by:

$$\frac{\partial\Psi_{t_0-\overline{a}(t_0)}(\overline{a}(t_0)-1,\Delta a(t_0),\kappa_{t_0-\overline{a}(t_0)})}{\partial\Delta a(t_0)}=0$$

Thus:

$$
\begin{aligned}
&\ln\left(\frac{p_{t_0}}{p_{t_0-1}}\left(\frac{1-\delta}{1+r_{t_0-1}}\right)\right)\\
&\left\{\begin{array}{l}\Delta a(t_0)\left((1-tr_{t_0})(1+\gamma)^{-\overline{a}(t_0)-\Delta a(t_0)+1}F(\kappa_{t_0-\overline{a}(t_0)},1)-\left(\frac{w_{t_0-1}}{p_{t_0-1}}\right)^{1-\Delta a(t_0)}\left(\frac{w_{t_0}}{p_{t_0}}\right)^{\Delta a(t_0)}\right)\\ -{x_{t_0-1}^f}^{1-\Delta a(t_0)}{x_{t_0}^f}^{\Delta a(t_0)}\end{array}\right\}\\
&+\left((1-tr_{t_0})(1+\gamma)^{-\overline{a}(t_0)-\Delta a(t_0)+1}F(\kappa_{t_0-\overline{a}(t_0)},1)-\left(\frac{w_{t_0-1}}{p_{t_0-1}}\right)^{1-\Delta a(t_0)}\left(\frac{w_{t_0}}{p_{t_0}}\right)^{\Delta a(t_0)}\right)\\
&-\Delta a(t_0)\left(\frac{w_{t_0-1}}{p_{t_0-1}}\right)^{1-\Delta a(t_0)}\left(\frac{w_{t_0}}{p_{t_0}}\right)^{\Delta a(t_0)}\ln\left(\frac{w_{t_0}/p_{t_0}}{w_{t_0-1}/p_{t_0-1}}\right)\\
&-{x_{t_0-1}^f}^{1-\Delta a(t_0)}{x_{t_0}^f}^{\Delta a(t_0)}\ln\left(\frac{x_{t_0}^f}{x_{t_0-1}^f}\right)\\
&-\left(\Delta a(t_0)\left(\frac{w_{t_0-1}}{p_{t_0-1}}\right)^{1-\Delta a(t_0)}\left(\frac{w_{t_0}}{p_{t_0}}\right)^{\Delta a(t_0)}+{x_{t_0-1}^f}^{1-\Delta a(t_0)}{x_{t_0}^f}^{\Delta a(t_0)}\right)\ln(1+\gamma)=0
\end{aligned}
\tag{8}
$$

It is easy to prove that in the neighborhood of the steady state and for small growth rates, (7) and (8) determine a unique integer value for $\overline{a}(t_0)$ and the real value included between 0 and 1 for $\Delta a(t_0)$ (proof available on request).

It is now possible to define the aggregate level of employment and production. At date t, the production structure available is characterized by the series: $\{n_{t-a}(1-d)^a, k_{t-a}\}$, where a is the age of the different production units in working order ($1 \le a \le \overline{a}(t_0)$), and by $\Delta a(t_0)$. Aggregate employment and production capacity are obtained by summing over these vintages[8]:

$$N_{t_0} = \sum_{a=1}^{\overline{a}(t_0)-1} n_{t_0-a}(1-\delta)^a + \Delta a(t_0) n_{t_0-\overline{a}(t_0)}(1-\delta)^{\overline{a}(t_0)} \tag{9}$$

[8] We could add the assumption that once a productive unit was scrapped, it cannot be put in use again. In this case, we should introduce the constraint $\overline{a}(t_0) \le \overline{a}(t_0 - 1) + 1$ in our optimization problem. The maximum number of available units of production can be called the *physical productive capacity*. In general this capacity will not be saturated, the exception being a strong unanticipated increase in demand at time t_0.

The creation of employment at time t_0 is n_{t0-1} and the destruction of employment is $-N_{t0} + N_{t0-1}$.

$$Y_{t_0} = A_{t_0} \sum_{a=1}^{\bar{a}(t_0)-1} (1+\gamma)^{-a} F(\kappa_{t_0-a}, 1) n_{t_0-a} (1-\delta)^a$$
$$+ \Delta a_{t_0} (1+\gamma)^{-\bar{a}(t_0)} F(\kappa_{t_0-\bar{a}(t_0)}, 1) n_{t_0-\bar{a}(t_0)} \quad (10)$$

III - CLOSURE AND EQUILIBRIUM OF THE MODEL

In this section we close the model in the simplest possible way. Then, we investigate how the current equilibrium of the economy is determined when its past and its expected future are known.

3.1. Aggregate demand and labor supply

The model is completed by adding three equations: the equilibrium of the goods market, the intertemporal arbitrage equation of a representative consumer and a wage curve.

Aggregate supply must be equal to total demand, which consists of real consumption, investment in equipment and other spending G_{t_0}.

$$Y_{t_0} = C_{t_0} + I_{t_0} c_{i t_0} + G_{t_0} \quad (11)$$

Other spending is financed by a production tax:

$$tr_{t_0} Y_{t_0} = G_{t_0} \quad (12)$$

In the rest of the paper, tr_{t_0} will be considered to be exogenous and G_{t_0} endogenous.

Consumption verifies the following Euler equation which assumes constant relative risk aversion (r) and time preference (β) of households. Current consumption depends on its expected level at the next period and on the real return on the risk-free asset between t_0 and $t_0 + 1$:

$$\left(C_{t_0+1} / ((1+\dot{n}) C_{t_0})\right)^\rho = \left(\frac{1+r_{t_0}}{1+\beta}\right) \frac{p_{t_0}}{p_{t_0+1}} \quad (13)$$

where $\dot{n}$ is the growth rate of the number of households and of the available labor force.

Finally, labor supply is defined as a wage curve linking the nominal wage in efficiency units to the price level, the wedge (reflecting the spread between the purchasing power of wages for workers and the effective labor cost for firms) and the employment rate (where $\overline{N}_{t_0}$ is the available labor force), which accounts for the effect of labor market conditions on wage bargaining.

$$\ln(w_{t_0}) = \varphi_0 + \ln(p_{t_0}) + \varphi_1 \ln(wedge_{t_0}) + \varphi_2 \ln(N_{t_0} / \overline{N}_{t_0}) \quad (14)$$

The general equilibrium of the model at date t_0 is determined by equations (1) to (14)[9]. Monetary policy pegs the nominal interest rate (r_{t_0}) and fiscal policy fixes the wedge ($wedge_{t_0}$) and the production tax rate (tr_{t_0}). Firing cost ($x^f_{t_0}$), the labor force ($\overline{N}_{t_0}$), and technical progress (A_{t_0}) are exogenous.

3.2. Neutrality, super-neutrality and the real interest rate

Two remarks must be made at this point. First, if we multiply the wage rate and the price for all lags and leads by any positive number, the equations of the model are still verified. Thus, the model presents the property of neutrality and can be rewritten in real terms.

Second, as new units start to produce at the following period, the real interest rate that firms take into account in their decision-making is defined by: $1+R_{t_0} = \dfrac{1+r_{t_0}}{1+pa_{t_0}}$, where $pa_{t_0} = \dfrac{p_{t_0+1}}{p_{t_0}}$ is the expected inflation rate. This real interest rate also governs the allocation of the households' consumption over time. Let us introduce the real wage rate in efficiency units $w_{t_0} = \dfrac{w_{t_0}}{p_{t_0}}$. Then, if we introduce the real interest rate and the real wage rate as new endogenous variables in the model, the nominal interest rate, the (expected) inflation terms and the price levels disappear from the equations. The model has the property of super-neutrality: changing the inflation rate changes the nominal interest rate by the same amount and has not effect on the real state of the economy. Actually, our model has a structure which is very similar to the structure of the neoclassical growth model. However, at the difference of the neoclassical growth model, the real interest rate appears with two lags in the scrapping equations.

Now, let us assume that the monetary authorities peg the nominal interest rate. So, the real interest rate is endogenous through the expected inflation pa_{t_0}. If the real interest rate did not appear with a lag in any equation, pa_{t_0} would be determined by the model. If we assume that simulations of the model start at time t_0, the initial price level p_{t_0} would be undetermined, the model having no nominal anchor. However, here the real interest rate appears with two lags, and we cannot assume that $pa_{t_0-1} = \dfrac{p_{t_0}}{p_{t_0-1}}$ is predetermined at time t_0 (or that πa_{t0-1} is known when we start the simulation). Thus, we introduce the observed inflation rate as a new endogenous variable:

[9] These equations written in the Troll syntax are available on request.

$$\pi_{t0} = \frac{p_{t0}}{p_{t0-1}} \tag{15}$$

with the consistency equation:

$$\pi_{t_0} = \pi a_{t_0+1} \tag{16}$$

Then, we can assume that $\boldsymbol{p}_{t_0-1}$ is predetermined at time t_0 (or that $\boldsymbol{p}_{t_0-1} = \dfrac{p_{t_0-1}}{p_{t_0-2}}$ is known when the simulation starts). We can eliminate all the lagged values of $\boldsymbol{p}a_{t_0}$, but we can show that the current equilibrium (i.e. the determination of the current values of the variables in function of their past and future values) is not defined. We can solve the problem if instead of pegging the nominal interest rate, we assume that the Central Bank follows the backward Taylor's rule:

$$1 + r_{t_0} = (1+\pi^*)(1+\beta)(1+\dot{n})^{-\rho} + 1.5(\pi_{t_0} - \pi^*) \tag{17}$$

where $p\ *$ is the target inflation rate and $(1+b)(1+\dot{n})^{-r}$ the steady state real interest rate.

Under this new assumption, the model can compute the path of all the variables, including the observed inflation rate $\pi_{t_0} = \dfrac{p_{t0}}{p_{t0-1}}$. So, the price level at the beginning of the simulation is determined by $p_{t_0} = \boldsymbol{p}_{t_0} p_{t_0-1}$, and we can compute all the paths of prices without ambiguity. The nominal anchor is given by the price level inherited from the past. The model presents an hysteresis for the level of prices and of all the other nominal variables[10].

3.3. Equilibrium of the model

With a putty-clay specification, aggregate supply adjusts to aggregate demand in two ways. First, investment flows provide new additional production capacities. Second, the profitability of available vintages commands the amount of production units that have to be scrapped.

[10] The problem of indeterminacy of the price level under a pure interest peg was first investigated by Sargent and Wallace (1975). It has brought a huge amount of literature with various solutions to bring a nominal anchor which solve this problem.

However, the production units created at date t_0 starts being productive at the beginning of period $t_0 + 1$. Hence, investment flows accrue to today's demand but to tomorrow's supply. Thus, at each period, supply can only adjust through the scrapping of old production units. As the technology of these vintages is set, their profitability only depends on the current real wages (in efficiency units). Hence, the scrapping rate of old production units is an increasing function of the current real wages.

Aggregate employment is defined by the amount of labor attached to each vintage of production still in working order. It increases with the age of the oldest profitable production units. Thus, aggregate employment decreases with real wages. As labor supply is represented by a wage curve according to which an increase in employment triggers an increase in wage claims, the equilibrium on the labor market determines real wages, aggregate employment and the age of the oldest units in working order.

Aggregate supply is obtained by summing all the vintages of production, profitable at current real wages. Current consumption is defined by the expected levels of next period consumption and the real interest rate. Then aggregate demand meets aggregate supply through investment flows. For any given capital intensity of the new production units, investment flows adjust through the number of units created.

The technological choices for the new units depend on the current and expected values for the real interest rate and real wages.

IV - CALIBRATION AND SIMULATION OF THE MODEL

4.1. The steady state and the calibration of the model

The basic time unit is one year. The model is calibrated on French data. Its steady state equilibrium replicates the 1994 French National Accounts for aggregate production, employment and investment (Y, N, I), as well as for the wage rate, the price level and the nominal interest rate (w, p, r).

The parameters to be evaluated are the characteristics of the production function (z and a), the scale factor of the wage equation j_0, and the risk premium m, which is introduced in an *ad hoc* way as the difference between the return on investment and the interest rate. Values of the unobserved endogenous variables must also be computed: the expected lifetime of the new units of production, their capital intensity and their number: T, k and n[11]. Actually, it can be shown that, with a slight approximation, the parameters and the unobserved variables only depend on the wage share in value added and the investment rate[12].

[11] At the steady-state $T + 1 = \overline{a} + \Delta a$.

[12] The equations of the steady state written with the Troll syntax, and the proof of the last assertion, are available on request.

The results are presented in the Table below. Note that investment flows correspond to material and equipment expenses made by firms. Dwellings, infrastructure and industrial buildings are integrated to the other expenses aggregate (G).

It is important to note that the expected scrapping date of the new units of production corresponds to the date of scrapping of the production units that have not gone bankrupt before. This date corresponds to a decision related to macroeconomic conditions (technical progress and wages). Nevertheless, the probability to go bankrupt for any reasons but global economic conditions is equal to d at each period. Hence, each new production unit depreciates at rate d until date T , when it is scrapped. The effective lifetime of a new unit can then be defined as:

$$D = \sum_{i=1}^{T} (1-d)^i = \left(\frac{1-d}{d}\right)\left(1-(1-d)^T\right)$$

Calibration of the model for France

National Accounts for the year 1994	
Share of wages in value added: $\dfrac{wN}{(1-tr)pY}$	0.81*
Tax rate: $tr = G/Y$	0.22
Investment rate: $I/[(1-tr)Y]$	0.14
Technical progress and population growth over the period 1980-1995	
Technical progress (per annum): g	0.022
Employment (manufacturing) growth rate (per annum)): $\dot{n}$	-0.002
Other parameters	
Probability to go bankrupt at the end of each period: d	0.04
Firing costs: x^f	5 months of wages
Installation costs: c_{i_0}	1.00
Elasticity of real wages to employment: j_2	0.30
Consumers' preferences: - relative risk aversion: r - time preference rate: b	 0.50 0.05
Short term real interest rate: R	0.064
Calibration of the supply-side	
Real return on investment: $R+m$	0.08
Lifetime of the new production units: T (years)	19.74
Effective lifetime of a new production unit: D (years):	13.28
Capital intensity: k	2.35
Job creation rate: n/N	0.074
Production function parameters (for s =0.99) a z	 0.22 1.60

Source: "Comptes et indicateurs économiques, rapport sur les comptes de la nation", various issues.

* This share might seem high relative to usual figures (e.g., Prigent 1999). In the model the « share of wages in value added » corresponds to the cost of labor (wages + contributions and tax on wages) relative to value added after tax on production. In 1994, the cost of labor was equal to 2 437 371 FRF (gross wages equal to 1 680 388 FRF) and the net value added was equal to 3 024 869 FRF (gross value added equal to 3 88 006).

4.2. Blanchard and Kahn's conditions and simulation method[13]

The model calibrated on French data was rewritten in reduced variables, and its linear approximation was computed around the reference steady state. Transforming the original variables to reduced variables is equivalent to give them a common trend of rate 0. We can also define expanded variables such that their common trend is equal to the highest balanced growth rate present in the model. The eigenvalues of the linear approximation of the model written in expanded variables are equal to the eigenvalues of the linear approximation of the model written in reduced variables, multiplied by the highest balanced growth rate plus one. The requirement for the model to converge in the long run to its balanced growth path is more severe when we consider expanded variables (stability in the expanded difference) than when we consider reduced variables (stability in the relative difference). The severity is intermediate when we consider the original variables of the model (which have different long run growth rates), but in this case, the coefficients of the linear approximation of the model depend on time, and the local conditions for the existence and the uniqueness of a solution, developed by Blanchard and Kahn (1980), do not apply.

The model has 39 non-redundant lead variables. When we consider the linear approximation with reduced variables, we have 39 eigenvalues with modulus larger than 1. The Blanchard and Kahn’s conditions are then verified, and there is a unique solution path of the reduced form model. Moreover, the largest eigenvalue less than 1 is equal to 0.932. Since the highest growth rate in the model is the real GDP growth rate, which is assumed to be equal to 1.9%, the Blanchard and Kahn’s conditions are also verified for the expanded form model (0.932*1.019<1). Hence, stability in expanded difference and stability in relative difference are verified. These two conditions are sufficient to ensure the local existence and uniqueness of a solution path for the model in original variables.

We simulate the model with a relaxation second order method implemented in Troll (Stack algorithm) over 150 periods. We assume that its original variables, which follow different trends, must converge in the long run to their balanced growth path.

4.3. Pseudo-hysteresis and replacement echo

Let us consider the linear approximation of the model with the reduced variables. We find that one of the eigenvalues smaller than 1 increases with the relative risk aversion of households. Actually, when r tends to infinity, equation (13) becomes:

$C'_{t_0+1} = C'_{t_0}$ where C'_{t_0} is the reduced consumption.

[13] The methodology used in this paragraph is developed in Laffargue (2000). All the computations were made under Troll, with the commands Lkroots and Stack.

So, for high relative risk aversion (r), this eigenvalue is close to 1, but remains below unity. Hence stability in the relative difference is satisfied. However, the Blanchard and Khan's conditions are not verified anymore for the expanded form model, because the previous eigenvalue time the long run real growth rate of the economy is larger than 1. For example, when $r = 5$, the highest eigenvalue with modulus less than 1 in the reduced form model is equal to 0.992. Thus, in the expanded form model, this eigenvalue becomes 0.992*1.019>1. This situation, when the Blanchard and Khan's conditions are verified for the linear approximation of the model written in reduced variables but not in expanded variables, is called pseudo-hysteresis by Laffargue (2000). In this case, we do not know if the model written in its original variables has a unique solution. Actually, when we try to simulate the model, the simulation algorithm diverges, which suggests, according to Boucekkine (1995), that there is no solution to the model.

Most of the eigenvalues smaller than 1 are complex, with periods equal to the expected lifetime of capital (19.30 years) and to its harmonics (9.60, 6.45, 4.87, 3.92, 3.28, 2.83, 2.48). All these eigenvalues have a magnitude of about 0.9. We have the same result with the smallest eigenvalues larger than 1: a period of 18.22 and its harmonics, and a magnitude of about 1.2. Both sets of eigenvalues represent the Fourier decomposition of the two cycles related respectively to a backward side of the model (scrapping) and to a forward side of the model (the building of new units). These cycles are well documented in the literature on vintage capital models, and are called replacement echo effect.

Let us simplify the model by assuming that firing costs are zero ($x^f = 0$), that the probability to go bankrupt is zero ($d = 0$), that population is constant ($\dot{n} = 0$) and that the production tax rate is zero ($tr = 0$). Then, when relative risk aversion r tends to 0, that is when the utility function of households becomes linear, the magnitude of both sets of eigenvalues tends to 1. Thus, we get the same result as Boucekkine, Germain and Licandro (1997) and Boucekkine, Germain, Licandro and Magnus (1998). When utility is a linear function of consumption, the replacement echo effect is permanent: the past path followed by capital accumulation reproduces eternally over time and we have a cyclical hysteresis mechanism in the model. When utility is strictly concave, the replacement echo dampens over time.

V - MEDIUM TERM CHANGES IN THE DISTRIBUTION OF INCOME IN FRANCE

5.1. Shock on the wage-setting relationship

We consider a permanent unanticipated change in the wage-setting relationship in France: the wage rate determined by the wage curve (14) is increased by 1%, all other things being kept equal[14]. This change may stem from institutional or political events that permanently shift the wage curve upward, such as an increase in the unions' power or more generous unemployment benefits which improve the outside opportunity of incumbent workers.

[14] This is made by an increase of the parameter j_0 .

In the steady-sate, the real interest rate is determined by consumers' preferences. Hence, the rate of return and the capital intensity of the production units are unchanged. Considering the factor cost frontier, the wage rate that firms are able to pay is also unchanged. Thus, the upward shift in the wage curve leads to a decrease in total employment and an increase in unemployment. Since labor is the bounding production factor in the long run, output decreases in the same proportion as employment. There is no change in the wage share in value-added in the long run, nor in the expected lifetime of capital.

However, the path to the new steady-state has interesting properties. The Appendix presents the results of this simulation (Charts A1 to A6). At the first period, real wages increase by a little less than 1% (Chart A4). The profitability of existing production units deteriorates. The age of the oldest units in working order decreases and employment and aggregate supply fall (Chart A2). Then, aggregate demand (Chart A1) adjusts downward with investment flows: less new units are created.

At time 2, the less numerous new units created at the previous period replace the old units in the set of vintages in working order. This substitution contributes to the fall in aggregate labor demand. Hence, labor demand can meet labor supply with a smaller fall in the age of the oldest unit in working order. This is mirrored in a smaller increase in real wages than in the previous period. Aggregate supply also falls with the replacement of the old units by the new ones. Once again lower investment flows adjust aggregate demand downward by the creation of vintages of smaller size.

As time goes on, the replacement of old vintages by smaller size ones intensifies and the increase in real wages needed to balance the labor market vanishes. Hence, after the initial upward shift, real wages come back progressively to their long run level. This trend leads investors to choose less capital-intensive technology for the units created during the transition path (Chart A3). According to the factor price frontier, real wages and real interest rate follow opposite trends in their convergence toward the long run (Chart A4).

Beyond the transitory capital-labor substitution, the driving force in the adjustment of the economy is the permanent decrease in investment flows, which feeds the progressive decline in GDP and employment.

Four interesting results deserve to be mentioned. First, when wages increase at the beginning, the scrapping of the oldest units decreases employment by a small amount, so the labor share in value-added increases (Chart A5). Over time, old units are substituted by new units which are less labor intensive, employment and wages decrease, and the labor share comes back to its initial level.

Second, as the consumption behavior is based on the permanent income theory, the progressive increase in the real interest rate is mirrored in a progressive decrease of consumption towards its depressed long run level. At the first period, consumption is however slightly higher than before the shock (Chart A1) due to the dominant effect of the fall in real interest rate (Chart A4).

Third, the replacement echo effect appears very clearly on the chart of investment (Chart A1). However, the graphic is a bit difficult to comment, precisely because the capital intensity of investment decreases over time.

Finally, the medium term adjustment of total employment is mostly driven by the fall in job creation (Chart A6). Job destruction occurs only at the first period.

5.2. Vintage capital and changes in the wage share in value-added in France since the 70s

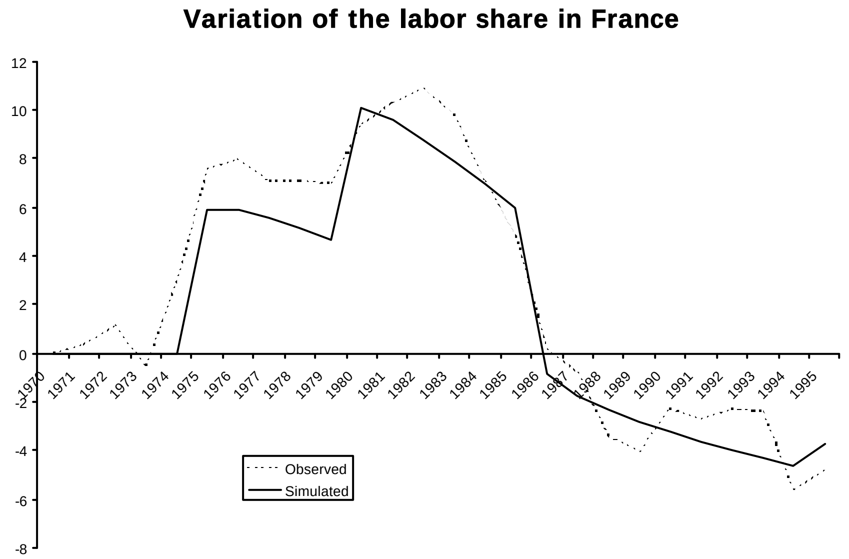
Recent economic research has emphasized the importance of medium term dynamics in continental Europe countries (Blanchard, 1997). Focussing on France, Caballero and Hammour (1998) present, in a vintage capital framework, an analysis of the changes in the wage share in value-added over the last 30 years by studying the impact of labor market institutions and regulations. Starting in 1970, the wage share in value-added is characterized by two jumps in 1974 and 1979, which altogether correspond to a 6 percentage points increase. Then, from the beginning to the late 80's, the wage share in value-added has decreased by around 14 percentage points, and has leveled off since then (Prigent, 1999).

Caballero and Hammour (1998) show that the changes in this share, as well as the steady increase in unemployment over the period, could be the outcome of the implementation, during the 70's and the early 80's, of more stringent employment regulations and more generous unemployment benefits. These changes, which first aimed at preventing the surge in layoffs and at dampening the fall in income of the unemployed following the two oil shocks, may well have eventually made permanent the upward shift in unemployment.

Here, we consider the ability of our model to replicate the same trends. Labor market reforms in France over the last three decades have been numerous and have concerned many areas. The analysis of their effect on the economy in the current framework requires to make gross approximations and to limit us to the major changes in institutions and regulations. Following Caballero and Hammour (1998), we retain three main shocks on the labor market. First, stronger constraints on dismissals (requirement of "real and serious" motives in 1973, administrative authorization for economically-motivated dismissals in 1975) were implemented in the middle of the 70's. They can be approximated by a shift in favor of workers in the bargaining over wages. Second, the Auroux labor laws of year 1982 mostly improved the local power of workers within the firm. Finally, some constraints on dismissals have been revised downward, notably the administrative authorization for economically-motivated dismissals was eliminated in 1986. These three shocks are implemented in the model as unanticipated shifts in the *wage curve* (the first two being upward shocks and the last on being a downward move). It is very difficult to calibrate the shock in the current framework since our *wage curve* summarizes many aspects of the bargain over wages. However, considering that each of the three shocks on the labor market corresponds to an *ex ante* shift equivalent to about 15% of real wages yields interesting results (see Chart)[15].

With this calibration, the vintage capital model is able to reproduce the major changes in the labor share in value added that the French economy experienced during the last 30 years. The first two shocks of the 70's explain the upward shift in this share over that decade. In the 80's, the medium term adjustment shows off: the wage-share in value added decreases toward its initial level. This trend is accelerated by the downward wage shift of the mid 80's, which eventually leads to a fall in the wage share in value-added slightly below its initial level during the late 80's and the first half of the 90's.

[15] The wage share in value added is taken from the OECD National Accounts. Prigent (1999) shows similar but less pronounced trends using INSEE National Accounts and various fields for firms (excluding large state-owned enterprises, or including individual enterprises). With these data, wage shifts of around 10% would be sufficient to provide the same fit as in Chart 1.



VI - CONCLUSION

This paper presents a macro-economic model assuming putty-clay investment and perfect foresight. Research on putty-clay technology was popular in the 70's (Adachi (1974), Britto (1969, 1970), Calvo (1976), Mizon (1974)) and was at the center of the analysis of the consequences of the oil shock on factor demands in the beginning of the 80's. However, it was eventually more or less abandoned for its lack of tractability, especially under the hypothesis of rational expectations. Since the mid 90s, from the works by Caballero and Hammour (1994) and Boucekkine et al. (1997), this research field has known a renewed interest for its ability to explain some major economic developments observed in industrialized countries over the last three decades. First, as putty-clay technology involves some stickiness in the production process, it enables to investigate properly the slow adjustment of production factors to shocks. Second, this framework also explicitly takes into account movements in job creation and job destruction related to economic obsolescence, replacement of productive capacity and expectations over the lifetime of the new units.

The originality of the model proposed here is that it is in discrete time whereas previous, recent works develop model in continuous time. Even under the strongest assumptions, it is almost impossible to derive the analytical solutions of continuous time models. The trick then usually consists in deriving discrete time formula from these models. Here, we prefer to start directly with a discrete time framework and use a second order relaxation algorithm to simulate the model. The traditional drawback of such a process was the presence of variables with long leads and long lags. However, progress in computation techniques has overcome these difficulties. For instance, by using the Stack algorithm implemented in Troll, the model can be easily solved.

The discrete time model has other advantages since it is possible to compute the eigenvalues of the dynamic system. First, this is useful to check the conditions of existence and uniqueness of a solution (Blanchard and Khan's conditions). More importantly, the analysis of the eigenvalues improves the understanding of the different dynamics in the economy. In particular, we can identify the echo effect that characterizes vintage capital models and the related dynamics of creation and destruction.

This kind of models has been proved useful to explain the medium-term movements in the distribution of income between production factors that putty-putty models lack. In particular, it illustrates quite well the change in the wage share in value-added in France during the last three decades. Further research could be realized to analyze more systematically the job creation and job destruction process, according to alternative assumptions on labor firing costs. In other words, we could investigate the extent to which these costs can prevent unprofitable old units from being scrapped and replaced by newly created units which embody new technology (as Cabballero and Hammour, 1994, showed with a continuous time model).

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APPENDIX

SIMULATION RESULTS

Chart A1: Aggregate demand

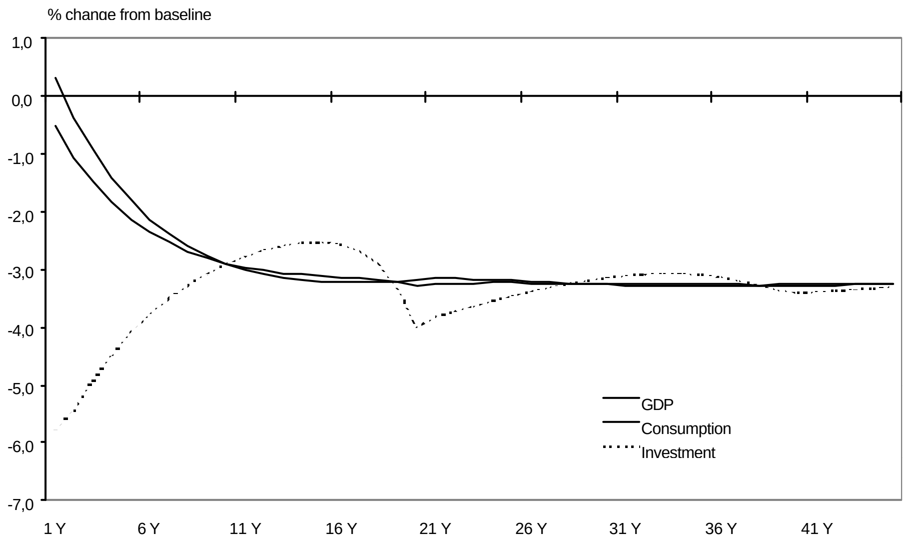

Chart A2: Aggregate supply

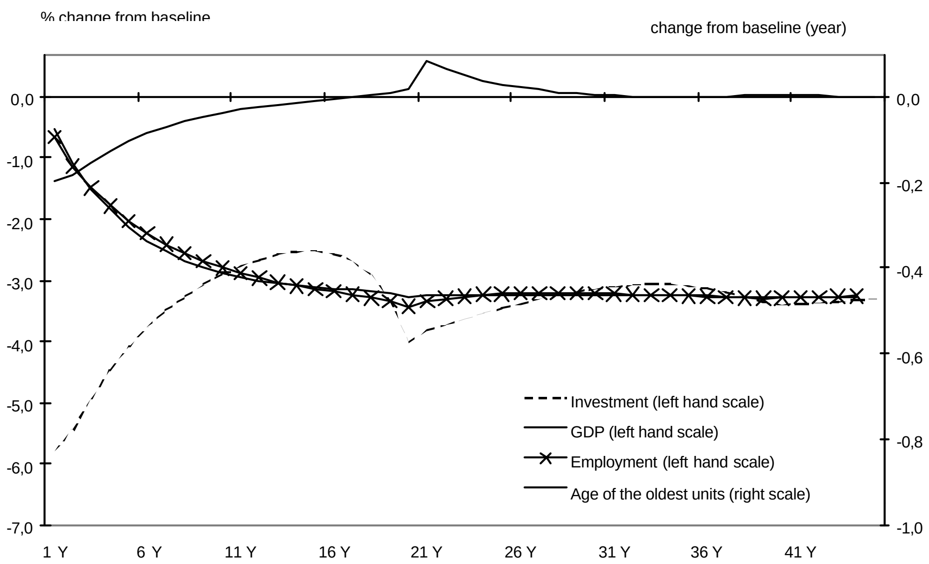

Chart A3: Technology choices for the new units

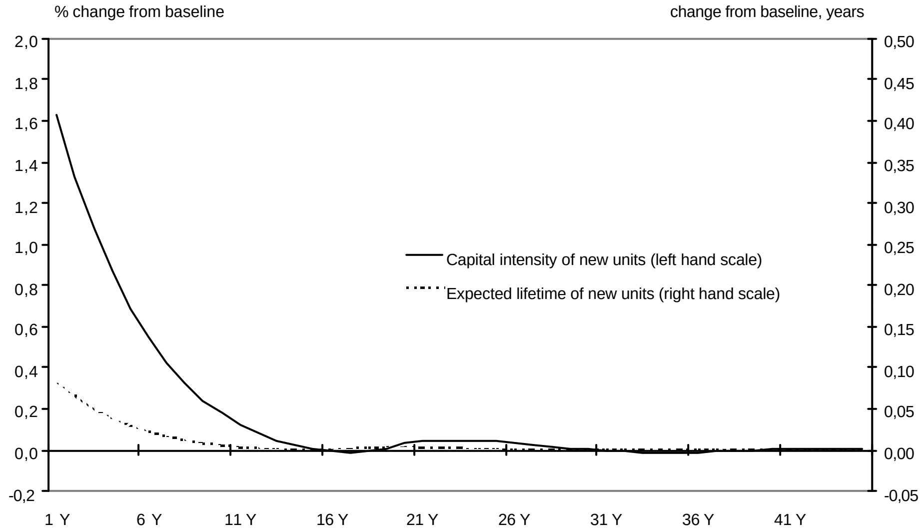

Chart A4: Real interest rate and wage rate

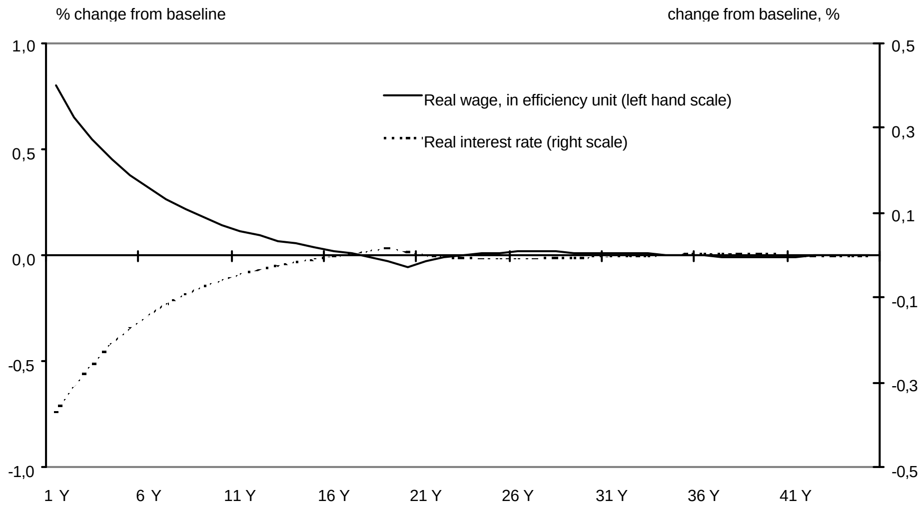

Chart A5: Labor share in value added and investment rate

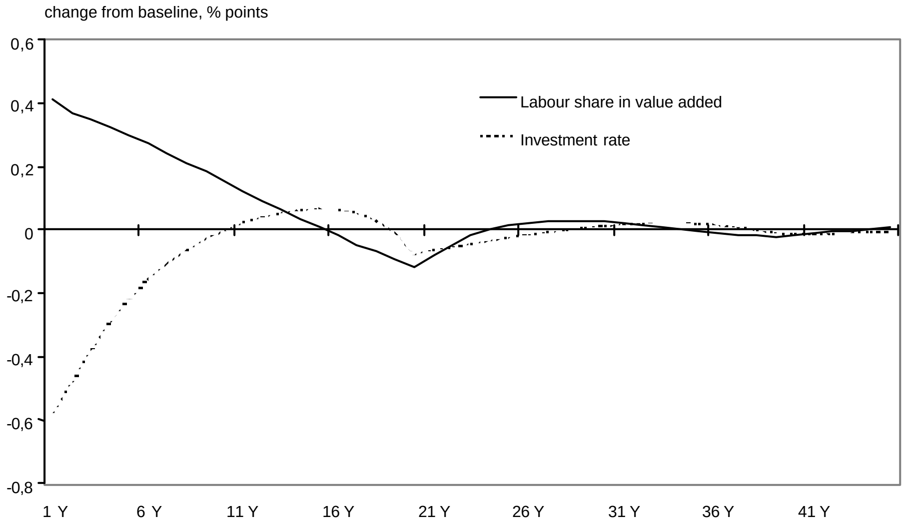

Chart A6: Job creation and job destruction

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